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2D DOA Estimation Method based on Channel State Information for Uniform Circular Array

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***Abstract*—In orthogonal frequency division multiplexing (OFDM) based communication systems multiple carriers having different frequencies are used to transmit different data at the same time. Complex values that describe attenuation on different subcarriers are called channel state information (CSI). This paper describes a novel method of two dimensional (2D) direction of arrival (DOA) estimation for uniform circular array (UCA) using CSI, available in transmitted OFDM signal. Firstly, using the fact that CSI among subcarriers comprises phase shift due to DOA and Time-of-Flight (ToF) number of antennas is virtually extended. Secondly, beamspace transform is applied to UCA's array manifold for CSI smoothing to virtually extend the number of observations. Finally, multiple signal classification (MUSIC) algorithm is applied on smoothed data for 2D DOA estimation. Comprehensive results and analysis are provided to show the superior performance of the proposed algorithm compared to the previous literature.**

I. Introduction

Estimation of direction of arrival (DOA) is a key task in a lot of fields like sonar, radar, seismology etc. for almost a century. In last few decades with development of wireless communications localizing mobile units became a problem too. Typical localizing schemes can be classified in three types: triangulation, scene analysis, and proximity [1]. In [2] authors proposed a novel approach called FILA. It is implemented on orthogonal frequency division multiplexing (OFDM) system using scene analysis. It exploits channel state information (CSI), a complex values that describe attenuation on every subcarrier, to build a propagation model and a fingerprinting system at the receiver. While in [3] a novel location signature CSI-MIMO incorporates CSI together with Multiple Input Multiple Output (MIMO) technology for fingerprinting. MIMO is a smart antenna technology that uses multiple antennas on the transmitting and receiving side, using which it is possible to apply DOA estimation, which is triangulation type, in wireless communications. ArrayTrack [4] is one of the systems which uses MIMO for DOA estimation. ArrayTrack's APs overhear the transmission and compute DOA of the transmitting user based on incoming frame. SpotFi [5] is another example of system that exploits MIMO for DOA estimation, but it also exploits CSI that is reported by commodity Wi-Fi card. Main disadvantage of [4] and [5] is that they use widely studied uniform linear array (ULA) for DOA estimation, which can provide only one dimensional angle estimates relative to the array axis. However, nowadays estimation of DOA in two dimensions (2D) is expected, and planar arrays are needed in this case. Recently, more and more advanced antenna structures have been developed to fulfill mobile communications requirement. And it is very likely that in the next $5th$ generation of mobile communications uniform circular array (UCA) antenna structure will be used at the base stations. Also, OFDM scheme is already deployed in 4G network at downlink, and at uplink SC-FDMA modulation scheme is used, which in fact is a special case of multiple carrier modulation. So it is of high interest to develop highly accurate 2D DOA estimation method using these features.

This paper introduces DOA estimation method in two dimensions, i.e. estimation of azimuth and elevation angles, based on CSI for uniform circular array. Received signal among antennas on UCA have different phase shifts due to incoming DOA relative to the origin of UCA. And CSI among subcarriers have different phase shifts due to different Time-of-Flights (ToF). Using this fact, number of antennas in array manifold is virtually extended. Later beamspace transform is applied to extend the number of observations, and finally DOA is estimated using multiple signal classification (MUSIC) [6] algorithm.

The rest of the paper is organized as follows. In Section II system model is described. Section III explains virtual extension of the number of antennas, while in Section IV extension of the number of snapshots/observations is explained. Simulation results and conclusion are presented in Section V and VI respectively.

II. System Model

A uniform circular array is assumed, i.e. N identical and omnidirectional antennas assumed to be uniformly distributed over circumference of radius R. A spherical coordinate system is used to represent azimuth and elevation angle of a plane wave impinging on antennas from the far-field. Array elements are situated on the xy plane, and array center is situated at the origin of the coordinate system. Azimuth angle is measured counterclockwise from the x axis and elevation angle down from the z axis. Fig 1 shows the geometry of the array. Output of the nth array element has phase difference $\psi_n = e^{jk_0 R sin\theta cos(\varphi-\gamma_n)}$ [7] relative to the origin, because nth array element is displaced from the x axis by an angle $\gamma_n = 2\pi n/N, n = 0, 1, \ldots, N-1$, where $k_0 = 2\pi/\lambda$ is a wavenumber, λ is a propagation speed of a planewave, φ and θ are azimuth and elevation angles, respectively. Assume that there are L plane waves impinging on the array from L distinct directions $(\theta_1, \varphi_1), \ldots, (\theta_L, \varphi_L)$. Using complex envelope notation [8] [9], array output vector with dimension $N \times 1$ can be expressed as

$$\vec{x} = \sum_{l=1}^{L} \vec{a}(\theta_l, \varphi_l) s_l + \vec{n} \tag{1}$$

where $s_l(t)$ is a complex envelope of an lth plane wave at the array center, $\vec{a}(\theta_l, \varphi_l)$ is a steering vector of the array towards lth plane wave's direction (θ_l, φ_l), and $\vec{n}(t)$ is a noise vector.

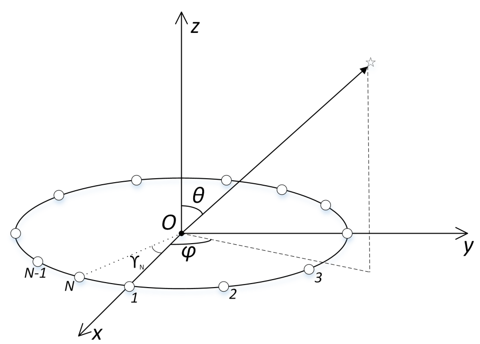

Fig. 1. UCA Structure.

Using matrix notation eq. (1) can be written as

$$\vec{x} = \mathbf{A}\vec{s} + \vec{n} \tag{2}$$

where $\vec{n}$ is a vector of additive white Gaussian noise (AWGN) [10], $\mathbf{A}(\Theta)$ is an $N \times L$ matrix of L steering vectors called array manifold that is written as

$$\mathbf{A} = \left[\vec{a}(\theta_1, \varphi_1), \ldots, \vec{a}(\theta_L, \varphi_L)\right] \tag{3}$$

and steering vector is written as

$$\vec{a}(\theta, \varphi) = \left[\psi_0, \ldots, \psi_{N-1}\right] \tag{4}$$

In eq. (2) $\vec{s} = [\alpha_1, \ldots, \alpha_L]$ is a vector of complex attenuations along L paths.

III. The CSI Processing

In OFDM system data is transmitted simultaneously over several subcarriers with different frequencies. So, in this case eq. (2) can be written for every subcarrier as

$$\mathbf{X} = [\vec{x}_1, \ldots, \vec{x}_K] = \mathbf{A}(\Theta)[\vec{s}_1, \ldots, \vec{s}_K] + \vec{n} = \mathbf{AS} + \vec{n} \tag{5}$$

where K is the number of subcarriers used by OFDM system. Matrix $\mathbf{X}$ that includes column vectors $\vec{x}_1, \ldots, \vec{x}_K$ denotes array output at every of the subcarriers, and matrix $\mathbf{S}$ in a similar fashion denotes complex attenuations of every path on every of the subcarriers. And because steering vectors don't change across closely spaced subcarriers [6] the array manifold $\mathbf{A}$ will not change too.

In OFDM system the attenuation and phase shift at each antenna introduced by every subchannel is represented by a complex CSI matrix. Also, in OFDM system signal is complex, i.e. it is a superposition of several sinusoids with different frequencies. Upon impinging on the array these sinusoids pass through the same path or, in other words, they travel with the same ToF, and obviously ToF introduces different phase shifts on every subcarrier (due to different frequencies). Thus, it is obvious that CSI values among different subcarriers have different phase shifts. In fact, column vectors $\vec{x}_1, \ldots, \vec{x}_K$ in eq. (5) correspond to columns in CSI matrix, and observation matrix $\mathbf{X}$ corresponds to the CSI matrix itself.

As an example, phase shift difference between two adjacent subcarriers in OFDM system is

$$\Omega(\tau_l) = e^{-j2\pi f_\delta \tau_l} \tag{6}$$

where f_δ is a frequency spacing between adjacent subcarriers and τ_l is a ToF of an lth path. To summarize, rows in the CSI matrix have phase shift difference due to DOA relative to the center of the UCA, and columns have phase shift difference due to ToF. Thus, we can rewrite CSI matrix in DOA and ToF dependence notation as

$$CSI = \begin{bmatrix} \Theta\Phi_0 & \Theta\Phi_0\Omega & \ldots & \Theta\Phi_0\Omega^{K-1} \\ \Theta\Phi_1 & \Theta\Phi_1\Omega & \ldots & \Theta\Phi_1\Omega^{K-1} \\ \vdots & \vdots & \ddots & \vdots \\ \Theta\Phi_{N-1} & \Theta\Phi_{N-1}\Omega & \ldots & \Theta\Phi_{N-1}\Omega^{K-1} \end{bmatrix} \tag{7}$$

there is azimuth angle dependence through $\Phi_n = e^{jcos(\phi-\gamma_n)}$, elevation angle dependence through $\Theta = e^{j\vartheta}$, and ToF dependence through Ω, where $\vartheta = k_0 R sin(\theta)$. Now, by stacking rows of the CSI matrix in a column, we construct modified CSI matrix

$$\widehat{CSI} = \vec{a}(\theta, \varphi, \tau)\vec{\alpha} \tag{8}$$

which in fact is a column vector, that has steering vector with $N \times K$ rows

$$\vec{a}(\theta, \varphi, \tau) = \left[\overbrace{\Theta\Phi_0, ..., \Theta\Phi_0\Omega^{K-1}}^{\text{First antenna}}, ..., \underbrace{\Theta\Phi_{N-1}, ..., \Theta\Phi_{N-1}\Omega^{K-1}}_{\text{Last antenna}}\right]^T \tag{9}$$

where $\vec{\alpha}$ is a vector of attenuations along L paths. And there are as many steering vectors as the number of sources, and modified array manifold $\widehat{\mathbf{A}}$ is constructed by stacking all steering vectors columnwise. Note, that the phase of the complex attenuation in eq. (2) has phase shift due to DOA and ToF absorbed into it, and will be different for every subcarrier [5]. But here, the phase is same for all the subcarriers and all the antennas. Thus, number of antennas in the modified $\widehat{CSI}$ matrix is virtually extended (for simplicity we will denote $\widehat{CSI}$ matrix as $\widehat{\mathbf{X}}$ matrix). Main task at this step is to extract useful information from the $\widehat{\mathbf{X}}$ matrix, i.e. azimuth and elevation angle information. MUSIC algotihm does this through eigendecomposition of the covariance matrix $\widehat{\mathbf{X}}\widehat{\mathbf{X}}^H$, that results in a signal and noise subspace. The first of the two main assumptions of the MUSIC algorithm is that the number of rows in the array manifold $\widehat{\mathbf{A}}$ is larger than the number of columns, i.e. number of antennas in the array is larger than the number of sources. Which is obviously true. And the second assumption is that the number of columns of $\vec{\mu}$ in eq. (8) is larger than the number of rows, which means that the number of the observations must be larger than the number of sources. Next section describes of CSI smoothing for UCA to extend the number of observations.

IV. CSI-UCA SMOOTHING

Before applying CSI smoothing to extend the number of the observations, we shall make some preprocessing on the CSI matrix. It is needed, because CSI smoothing method in [5] is based on the Vandermonde property of the array manifold of the uniform linear array. Whereas in the case of UCA array manifold is not a Vandermonde matrix. This preprocessing is based on transforming the actual array manifold into a *virtual* one which will have Vandermonde property by a beamspace transformation.

First, we shall describe beamspace transformation for the case where the signal impinging on the array is not complex, i.e. there is only one carrier sinusoid. Suppose we have UCA output in matrix notation as below

$$\mathbf{X} = \mathbf{A}\mathbf{S} \tag{10}$$

where $\mathbf{X}$ is an $N \times M$ UCA output matrix, $\mathbf{A}$ is an $N \times L$ array manifold matrix, $\mathbf{S}$ is an $L \times M$ signal sample matrix, and M is the number of samples. Now we can transfer arraymanifold $\mathbf{A}$ from element space to the beamspace by premultiplying it with a beamformer $\mathbf{F}$

$$\widetilde{\mathbf{X}} = \mathbf{F}^H\mathbf{X} = \widetilde{\mathbf{A}}\mathbf{S} = \sqrt{N}\mathbf{J}_\zeta\vec{\nu}(\phi) \tag{11}$$

where $\widetilde{\mathbf{A}} = \mathbf{F}^H\mathbf{A}$ is a transformed *virtual* array manifold. Here azimuthal dependence is through the vector

$$\vec{\nu}(\phi) = [e^{-jh\phi}, \ldots, e^{-j\phi}, e^{j0}, e^{j\phi}, \ldots, e^{jh\phi}]^T \tag{12}$$

and elevation angle dependence is through the matrix of Bessel functions

$$\mathbf{J}_\zeta = diag[\mathbf{J}_h(\zeta), \ldots, \mathbf{J}_1(\zeta), \mathbf{J}_0(\zeta), \mathbf{J}_1(\zeta), \ldots, \mathbf{J}_h(\zeta)] \tag{13}$$

where $\zeta = \frac{2\pi R}{\lambda} sin(\theta)$, and h is a highest order mode that can be excited by the aperture at a reasonable strength [7]. We omit explanation of how beamformer and phase mode is derived for brevity, but reader can find comprehensive information of it in [11], [12] and [7].

In the complex signal case beamspace transformation is made in a similar way, we premultiply CSI matrix by a beamformer $\mathbf{F}^H$, and get transformed CSI matrix

$$\mathbf{F}^H\mathbf{X} = \sqrt{N}\mathbf{J}_\zeta \begin{bmatrix} e^{-jh\phi} & e^{-jh\phi}\Omega & \ldots & e^{-jh\phi}\Omega^{K-1} \\ \vdots & \vdots & \ddots & \vdots \\ e^{jh\phi} & e^{jh\phi}\Omega & \ldots & e^{jh\phi}\Omega^{K-1} \end{bmatrix} \tag{14}$$

It can be seen that after transformation besides azimuthal and elevation angle dependence, there is still ToF dependence among columns too. This way it is still possible to extend the number of antennas to get modified matrix $\widehat{\mathbf{X}}$ as it was is described in the previous section.

CSI smoothing is a mathematical trick [5], that is easier to explain using the following example. Assume 1 is the highest phase mode and that there are 5 subcarriers. Then CSI matrix after transformation can be written as

$$\widetilde{CSI} = \begin{bmatrix} e^{-j\phi} & e^{-j\phi}\Omega & e^{-j\phi}\Omega^2 & e^{-j\phi}\Omega^3 & e^{-j\phi}\Omega^4 \\ 1 & \Omega & \Omega^2 & \Omega^3 & \Omega^4 \\ e^{j\phi} & e^{j\phi}\Omega & e^{j\phi}\Omega^2 & e^{j\phi}\Omega^3 & e^{j\phi}\Omega^4 \end{bmatrix} \tag{15}$$

Now assuming there is two sources, this transformed $\widetilde{CSI}$ matrix can be divided into two subarrays as

$$\begin{bmatrix} \tilde{csi}_{1,1} & \tilde{csi}_{2,3} \\ \tilde{csi}_{1,2} & \tilde{csi}_{2,4} \\ \tilde{csi}_{1,3} & \tilde{csi}_{2,5} \\ \tilde{csi}_{2,1} & \tilde{csi}_{3,3} \\ \tilde{csi}_{2,2} & \tilde{csi}_{3,4} \\ \tilde{csi}_{2,3} & \tilde{csi}_{3,5} \end{bmatrix} = \begin{bmatrix} e^{-j\phi_1} & e^{-j\phi_2} \\ e^{-j\phi_1}\Omega_1 & e^{-j\phi_2}\Omega_2 \\ e^{-j\phi_1}\Omega_1^2 & e^{-j\phi_2}\Omega_2^2 \\ 1 & 1 \\ \Omega_1 & \Omega_2 \\ \Omega_1^2 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_1 e^{j\phi_1}\Omega_1^2 \\ \alpha_2 & \alpha_2 e^{j\phi_2}\Omega_2^2 \end{bmatrix} \tag{16}$$

Columns in the first matrix on the right hand side are the steering vectors, and α_1 and α_2 are complex attenuations along two paths. By weighing steering vectors by complex attenuations α_1 and α_2 we get first subarray on the left hand side of the eq. (16). The second subarray absorbs scaling factor $e^{j\phi}\Omega^2$ and we can write transformed CSI values of this second subarray by weighing the steering vectors by the modified complex attenuation $\alpha e^{j\phi}\Omega^2$. Thus, values in two subarrays are a linear combination of the same vectors (steering vectors) and the vector of complex attenuations of the first subarray is linearly independent of the vector of complex attenuations of the second subarray. This way the number of antennas and the number of independent observations is virtually extended, and MUSIC algorithm can be straightly applied to the transformed and smoothed CSI matrix.

V. SIMULATION RESULTS

First, we perform simulation of virtually extending number of antennas only. We assume to have UCA with 3 omnidirectional antennas only, with $\frac{\lambda}{2}$ element spacing. DOAs of two far

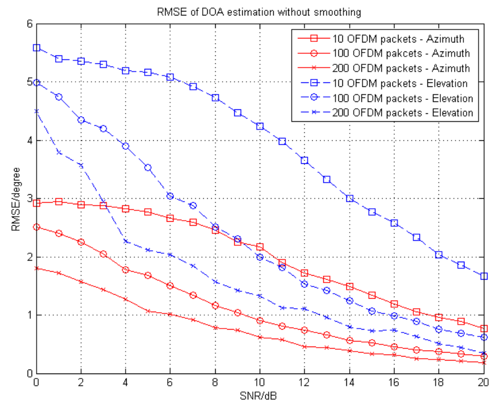

Fig. 2. RMSE of 1000 Monte Carlo simulations without smoothing

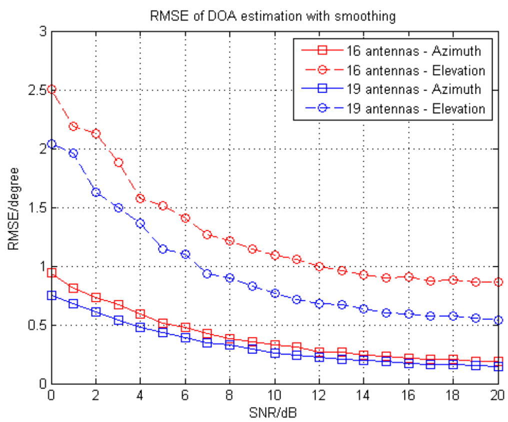

Fig. 3. RMSE of 1000 Monte Carlo simulations with smoothing

field sources are $(\theta_1, \varphi_1) = (90°, 51°), (\theta_2, \varphi_2) = (78°, 72°)$. As it has been previously mentioned in section III, the number of observations must be larger than the number of sources. But, if we assume extending the number of antennas only, then our observation matrix $\mathbf{S}$ has only one column, which means we have only one independent observation. To overcome this problem we assumed having CSI values of 10, 100 and 200 consecutive OFDM packets received at the UCA, during wich sources are assumed to be stationary. Root mean square error (RMSE) of DOA estimation is shown in fig 2.

Next, we perform simulations of CSI smoothing technique. For this case we assume UCA with 16 and 19 omnidirectional antennas, because according to [7] for beamspace transformation UCA must have at least twice the highest phase mode number of antennas. DOAs of incoming signals are set same as in the previous simulation. First source were assumed to have $10ns$ ToF, and secound source's ToF $90ns$. The results of 1000 Monte Carlo trials for azimuth and elevation angles are shown in fig. 3.

VI. Conclusion

We proposed a novel method of 2D DOA estimation based on CSI in OFDM for circular array. This method extracts information about the azimuth and the elevation angles, thus realising highly accurate two dimensional DOA estimation. First, by noting the fact that phase shift information due to ToF of a signal is included in complex attenuations of subcarriers with different frequencies, the number of antennas in the array was virtually extended. Next, by using phase mode exciation method, CSI values were transformed from element space to beamspace, to extend the number of independent observations, and MUSIC algorithm was applied. Several simulations were performed to verify the effectiveness and accuracy of the proposed method. Provided results show that in case of only virtually extending the number of antennas, although much smaller number of antennas were used, proposed method still achieved good results. Also, CSI-UCA smoothing using one OFDM packet can achieve RMSE of less than one degree at higher SNRs.

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